Stochastic giant resonance

Jing-hui Li
Faculty of Science, P.O. Box 58, Ningbo University, Ningbo 315211, China
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The model of an electric circuit with dichotomous resistance is investigated. It is shown that the dichotomous resistance can induce a phenomenon of stochastic giant resonance for the signal-to-noise ratio (SNR) as a function of input signal frequency. This phenomenon is a direct consequence of the existence of a zero noise power spectrum, with a nonzero signal power spectrum for the same parameters. In addition, two kinds of the usual stochastic resonance phenomena have been obtained for the SNR. One is as a function of the correlation time of the asymmetric dichotomous resistance; the other is as a function of the input signal frequency.

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The phenomenon of stochastic resonance (SR) caused by the appearance of noise has attracted considerable interest recently due to its many applications in biology, physics, chemistry, and other scientific fields [1–8]. Not only is the number of publications on SR phenomenon growing steadily, but much extension of the conception of SR has appeared, such as double SR [2], stochastic multiresonance [3], coherence resonance (generated in the system without an external force) [4], quantum SR [5], control of SR [6], autonomous SR [7], aperiodic SR [8], etc.

In the present paper, we will study a model of an electric system with a dichotomous resistance and subject to an input periodic-voltage source and a constant-voltage source. Some unusual and usual behaviors will be reported: The dichotomous resistance can induce a phenomenon of stochastic giant resonance for the signal-to-noise ratio (SNR) as a function of input signal frequency. In addition, two kinds of usual stochastic resonance phenomena can be obtained for the SNR. One is as a function of the correlation time of the asymmetric dichotomous resistance; the other is as a function of the input signal frequency.

We wish to analyze the dynamic behavior of the circuit system schematically depicted in Appendix A of our recent publication [9] (refer to Fig. 6 in Ref. [9]). Now, all its elements are conventional except for a dichotomous resistance $r(t)$, which fluctuates between two values r_a and r_b with mean waiting times t_a and t_b, respectively. This can be achieved by inserting into the circuit a point contact whose conductance is controlled by an asymmetric two-level system tunneling incoherently between the two states with rates $\gamma=1/t_a$ and $\gamma'=1/t_b$ [10,11]. Accordingly, the fluctuations in the point-contact resistance can be modeled as a stationary Markovian dichotomic process (telegraphic noise).

The dynamics of the circuit is governed by the stochastic differential equation

$$L\frac{di}{dt}+[R+r(t)]i=a\sin(\omega t)+U_0, \tag{1}$$

where $r(t)\in r_a,r_b$ $(r_a>r_b>0)$ is the dichotomous resistance which is a telegraphic noise, L the electric inductance (constant), R the electric resistance (constant), U_0 the constant voltage, and $a\sin\omega t$ an ac oscillatory voltage. For generality, we assume that $\langle r(t)\rangle=(r_a\gamma+r_b\gamma')/(\gamma+\gamma')=g_0$, in which g_0 is a constant. The covariance function of the asymmetric dichotomous noise $r(t)$ is $\langle r(t),r(t')\rangle=\langle r(t)r(t')\rangle-\langle r(t)\rangle^2=D\lambda\exp[-\lambda|t-t'|]$. Here $\lambda=(\gamma+\gamma')$ is the reverse of the correlation time τ of the asymmetric dichotomous noise, and the definition of the strength of the asymmetric dichotomous noise is $D=(1/2)\int_{-\infty}^{+\infty}\langle r(\tau),r(0)\rangle d\tau=[-r_ar_b+(r_a+r_b)g_0-g_0^2]/\lambda=(r_a-r_b)^2\gamma\gamma'/(\gamma+\gamma')^3$. We can find that the noise strength D is not independent, but is connected with the correlation time $\tau=1/\lambda$, g_0, and $r_{a,b}$ of the noise, or with r_a, r_b, γ, and γ'.

In Appendix A of Ref. [9], we discussed the same circuit. But only under the condition that $\gamma'=k\gamma$, $r(t)=-\xi(t)+g$ in which $\xi(t)$ takes values $-E$ and kE, $E=(r_a-r_b)/(1+k)$, and $g=(kr_a+r_b)/(1+k)$, Eq. (1) in this paper [or Eq. (A1) in Ref. [9]] can become Eq. (1) investigated by us in Ref. [9]. Otherwise, Eq. (1) in this paper [or Eq. (A1) in Ref. [9]] is quite different from Eq. (1) studied by us in Ref. [9]. So the main result which we will obtain in this paper—i.e., a phenomenon of stochastic giant resonance—could not be found for Eq. (1) studied by us in Ref. [9]. In addition, the investigation made in Ref. [9] is about the usual stochastic resonance, while the present study is mainly focused on the phenomenon of stochastic giant resonance (it is a *bona fide* resonance).

In order to make the calculation conveniently, we assume $r(t)=\eta(t)+g$, where $\eta(t)$ is a dichotomous noise, which takes two values c and $-c$, g is a constant, the transition rate from c to $-c$ is γ for $\eta(t)$, and the reverse transition rate is γ'. Taking the ensemble average of $r(t)=\eta(t)+g$, we can obtain $\langle\eta(t)\rangle=g_0-g$. We can also derive the correlation function of $\eta(t)$: $\langle\eta(t)\eta(t')\rangle=(g_0-g)^2+D\lambda\exp[-\lambda|t-t'|]$. Using the relations between the dichotomous noise $\xi(t)$ and the dichotomous noise $\eta(t)$, we can get $c=(r_a-r_b)/2$ and $g=(r_a+r_b)/2$. Substituting $r(t)=\eta(t)+g$ into Eq. (1), we get

$$\frac{di}{dt}=-\frac{(R+g)}{L}i-\frac{\eta(t)}{L}i+\frac{a}{L}\sin(\omega t)+\frac{U_0}{L}. \tag{2}$$

Multiplying $i(t')$ on both sides of Eq. (2) and taking the average over noise after that, we can obtain

$$\frac{d\langle i(t)i(t')\rangle}{dt}=-\frac{(R+g)}{L}\langle i(t)i(t')\rangle-\frac{\langle\eta(t)i(t)i(t')\rangle}{L}i+\frac{a}{L}\sin(\omega t)\times\langle i(t')\rangle+\frac{U_0}{L}\langle i(t')\rangle. \tag{3}$$

We can find that there is one new correlation factor

$\langle\eta(t)i(t)i(t')\rangle$ to appear in this equation. To solve the equation, we will make use of the well-known "formula of differentiation" [12] proposed by Shapiro and Loginov and extensively used in Ref. [13]. For the form of $\langle\eta(t)i(t)\rangle$, the formula reads [14]

$$\frac{d\langle\eta(t)i(t)i(t')\rangle}{dt} = -\left(\lambda + \frac{(R+g)}{L}\right)\langle\eta(t)i(t)i(t')\rangle + [\lambda(g-g_0) - c^2/L]\langle i(t)i(t')\rangle + \left(\frac{a}{L}\sin\omega t + \frac{U_0}{L}\right)\times\langle i(t')\eta(t)\rangle, \tag{4}$$

in which we have used $\eta^2(t)=c^2$ and $\langle\eta(t)\rangle=g_0-g$.

From Eqs. (3) and (4), we can get the expression of $\langle i(t)i(t')\rangle$ in the limit of $t\to\infty$. Then, let $|t-t'|=\tau$ ($t'=t+\tau$ and $t'=t-\tau$, respectively). Because the correlation function $\langle i(t)i(t\pm\tau)\rangle$ depends on both times t and τ, we should take the average about t within a period of $2\pi\omega^{-1}$. After that, we can get

$$\langle\langle i(t)i(t\pm\tau)\rangle\rangle_t = w_1\cos\omega\tau + w_2\sin\omega\tau + w_3\cos\omega\tau e^{-\lambda|\tau|} + w_4\sin\omega\tau e^{-\lambda|\tau|} + G_5 + G_6 e^{-\lambda|\tau|}, \tag{5}$$

where w_1, w_2, w_3, w_4, G_5, and G_6 are the functions of λ, U_0, L, R, a, g_0, ω, r_a, and r_b (since the expression of them are very long, I give the expression of them in Appendix A).

The power spectrum is

$$\begin{aligned} S(\Omega) &= \langle S(\Omega)\rangle_t + \langle S(-\Omega)\rangle_t \\ &= N_1\delta(\Omega) + N_2\delta(\Omega-\omega) + N_3 \\ &= 2G_5\frac{1}{2\pi}\delta(\Omega) + \frac{w_1}{2\pi}\delta(\Omega-\omega) + \frac{w_3}{2\pi}\left[\frac{2\lambda}{\lambda^2+(\Omega-\omega)^2} + \frac{2\lambda}{\lambda^2+(\Omega+\omega)^2}\right] + \frac{1}{\pi}\frac{\lambda G_6}{\lambda^2+\Omega^2}, \end{aligned} \tag{6}$$

in which $\langle S(\Omega)\rangle_t = \int_{-\infty}^{\infty}\langle\langle i(t)i(t\pm\tau)\rangle\rangle_t \exp(-i\Omega\tau)d\tau$.

Notice that the spectrum (6) divides naturally into three parts: the zero-frequency output that is a δ function at the zero frequency, the signal output that is a δ function at the signal frequency, and the broadband noise outputs that are three Lorentzian bumps centered at $\Omega=0$, $\Omega=-\omega$, and $\Omega=\omega$, respectively.

So the SNR can be obtained as

$$\mathrm{SNR} = \left|\frac{N_2(\omega)}{N_3(\omega,\Omega)}\right|_{\Omega=\omega} = \left|\frac{w_1}{\frac{2\lambda G_6}{\lambda^2+\omega^2} + w_3\left(\frac{2}{\lambda} + \frac{2\lambda}{\lambda^2+4\omega^2}\right)}\right|. \tag{7}$$

Below let us first analyze the power spectrum of the noise in Eqs. (6) and (7). It is

$$S_N = N_3(\omega,\Omega)|_{\Omega=\omega} = \frac{\lambda G_6}{\pi(\lambda^2+\omega^2)} + \frac{w_3}{\pi}\left(\frac{2}{\lambda} + \frac{2\lambda}{\lambda^2+4\omega^2}\right). \tag{8}$$

Let $S_N=0$; from Eq. (8), we can get

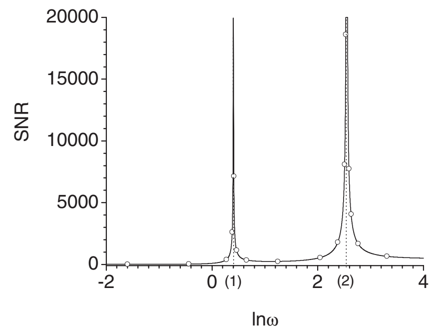

FIG. 1. The SNR versus $\ln(\omega)$ with $U_0=0.05$, $R=1$, $\lambda=2.71828$, $r_a=2$, $r_b=0.2$, $g_0=0.3$, $L=0.5$, and $a=0.2$ in dimensionless form. The circle points are the results of numerical simulations with the above fixed parameters.

$$a_{10}\omega^{10} + a_8\omega^8 + a_6\omega^6 + a_4\omega^4 + a_2\omega^2 + a_0 = 0, \tag{9}$$

in which a_{10}, a_8, a_6, a_4, a_2, and a_0 are functions of λ, U_0, L, R, a, g_0, r_a, and r_b (I give the expression of them in Appendix B). If Eq. (9) has positive real roots for some values of the given parameters λ, U_0, L, R, a, g_0, r_a, and r_b, the noise power spectrum will become zero when the frequency of the signal equals these positive real roots of Eq. (9). Then, when ω is in the vicinity of these positive real roots of Eq. (9), with the nonzero value of the power spectrum of the signal [i.e., $N_2(\omega)$ in Eq. (7)] for the same parameters, we can get a phenomenon of huge resonance for the SNR. For convenience, in this paper, we call this phenomenon "stochastic giant resonance."

In Fig. 1, we plot the SNR versus $\ln\omega$, with $U_0=0.05$, $R=1$, $\lambda=2.71828$, $r_a=2$, $r_b=0.2$, $g_0=0.3$, $L=0.5$, and $a=0.2$, in dimensionless form. The parameters values chosen in Fig. 1 should satisfy Eq. (9) with some real roots [this can be got by solving Eq. (9) with different parameters values] and nonzero signal power spectrum for the same parameters. This figure shows the emergence of the stochastic giant resonance phenomenon. For the given parameters $U_0=0.05$, $\lambda=2.71828$, $R=1$, $r_a=2$, $r_b=0.2$, $g_0=0.3$, $L=0.5$, and $a=0.2$, Eq. (9) has two positive real roots [the signal power spectrum is now nonzero, which can be easily got by substituting these parameters values into $N_2(\omega)$ in Eq. (6)]. They are $\omega_1 \doteq 1.49$ and $\omega_2 \doteq 12.81$, which corresponds to the points (1) and (2) in Fig. 1, respectively. So, for the fixed parameters $U_0=0.05$, $\lambda=2.71828$, $R=1$, $r_a=2$, $r_b=0.2$, $g_0=0.3$, $L=0.5$, and $a=0.2$, when $\omega=\omega_1$ or $\omega=\omega_2$, the noise power spectrum is zero and the signal-to-noise ratio tends to infinity. Then, when ω is in the vicinity of ω_1 or ω_2, we can get the phenomenon of stochastic giant resonance. Now it should be mentioned that, with the emergence of the stochastic giant resonance in Fig. 1, the amplitude of the intensity of the output signal [i.e., $\langle i(t)\rangle$] does not in turn shows a maximum [$\langle i(t)\rangle$ is easily got by solving a equation which is got by averaging the two sides of Eq. (2) and using the well-known "formula of differentiation" [12]].

Except for the phenomenon of stochastic giant resonance, the SNR for Eq. (1) can exhibit two kinds of the usual stochastic resonance phenomena. One is as a function of the

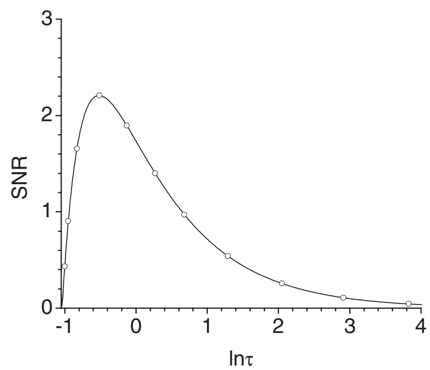

FIG. 2. The SNR versus $\ln(\tau)$ with $U_0=0.5$, $R=1$, $\omega=2.3$, $r_a=1$, $r_b=0.2$, $g_0=0.5$, $L=0.5$, and $a=0.2$ in dimensionless form. The circle points are the results of numerical simulations with the above fixed parameters.

correlation time of the asymmetric dichotomous noise (see Fig. 2). The other is as a function of the input signal frequency (see Fig. 3). From Figs. 2 and 3, we can see that the dichotomous noise and input periodic signal play two roles. On the one hand, they stimulate coherent motion with an increase of the asymmetry of the system; this motion does not exist in the absence of the noise and input periodic signal. On the other hand, they reduce the coherent motion with a decrease of the asymmetry of the system. The competition of these two opposite roles leads to the phenomenon of resonance. When the frequency of the input signal is equal to the intrinsic frequency of the system [16], the phenomenon of resonance appears [1]. To verify this, we have made some numerical calculations for the system frequency controlled by noise when the resonance occurs in Figs. 2 and 3. Numerical calculations show that the phenomenon of resonance appears when the signal frequency equals the system frequency controlled by the noise (in Fig. 2, with the appearance of resonance, the system frequency $f \doteq 0.33$, now the signal frequency $f_s=\omega/2\pi=2.3/2\pi \doteq 0.33$; in Fig. 3, with the emergence of resonance, the system frequency $f \doteq 0.32$, now the signal frequency $f_s=\omega/2\pi=2/2\pi \doteq 0.32$). So the parameters values chosen in Figs. 2 and 3 should satisfy the

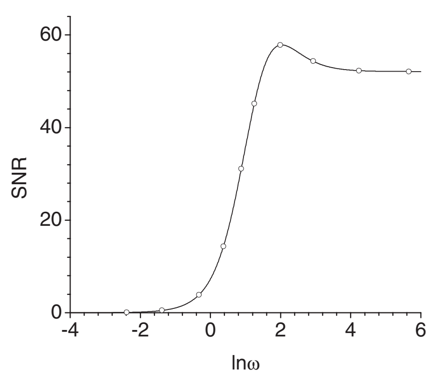

FIG. 3. The SNR versus $\ln(\omega)$ with $U_0=0.15$, $R=1$, $\lambda=2.71828$, $r_a=2$, $r_b=0.2$, $g_0=0.3$, $L=0.5$, and $a=0.2$ in dimensionless form. The circle points are the results of numerical simulations with the above fixed parameters.

condition that, by controlling the noise, there will be one point in the x axis (the x axis in Fig. 2 is the τ axis; the x axis in Fig. 3 is the ω axis) in which the system frequency is equal to the signal frequency (this can be obtained by suitably selecting the parameters values). With the appearance of the usual stochastic resonance in Figs. 2 and 3, the amplitude of the output signal [i.e., $\langle i(t)\rangle$] correspondingly represents a maximum [$\langle i(t)\rangle$ can be easily got by solving a equation obtained by averaging Eq. (2) and using the well-known "formula differentiation" [12]].

The results plotted in Figs. 1–3 are the results of a long algebraic calculation. A comparison with numerical simulations would help appreciate the correctness of the algebra. So, in Figs. 1–3, we give some results of the numerical simulation (see the circle points in these figures) [15].

Dichotomous noise contains as limits both white Gaussian noise and white shot noise; below, we consider the two limits. First, we consider the white-Gaussian-noise limit. In the limit of $\gamma,\gamma' \to \infty$, i.e., $\lambda \to \infty$, r_a and $r_b \to \infty$, and $D=[-r_a r_b+(r_a+r_b)g_0-g_0^2]/\lambda=\text{const}$ our asymmetric dichotomous noise becomes Gaussian white noise. Now, considering $\lambda \to \infty$, $r_a \to \infty$, $r_b \to \infty$, and $D=[-r_a r_b+(r_a+r_b)g_0-g_0^2]/\lambda=\text{const}$, constant, we can get the amplitude of the average electric current (i.e., the amplitude of the output signal): $A=(a/L)\sqrt{1/\{\omega^2+[D/L^2-(R+g)/L]^2\}}$. The white-shot-noise limit is $(r_a/r_b) \to \infty$, $\gamma' \to \infty$, and $r_b=\text{const}$, which is equivalent to the limit of $\lambda \to \infty$, $(r_a/r_b) \to \infty$, $D=\text{const}$, and $r_b=\text{const}$. Now, by using $\lambda \to \infty$, $(r_a/r_b) \to \infty$, and $[-r_a r_b+(r_a+r_b)g_0-g_0^2]/(r_b\lambda)=D/r_b$ with $D=\text{const}$ and $r_b=\text{const}$, the amplitude of the average electric current can be obtained as $A=0$. So it is clear that, in the limits of Gaussian white noise and Poissonian shot white noise, no phenomena of usual stochastic resonance appear for Eq. (1). Similarly, by calculating the SNR in the limits of Gaussian white noise and Poissonian shot white noise, we cannot find the phenomenon of stochastic giant resonance to occur.

In addition, the results shown in Figs. 1–3 are those for the case of $k \neq 1$—i.e., the asymmetry case for the dichotomous resistor. If the noise is not asymmetric—i.e., the noise has the same transition rates to the both states ($k=1$)—as long as Eq. (9) has positive real roots (with the nonzero value of the power spectrum of the signal for the same parameters), we can get the stochastic giant-resonance phenomenon as reported above; now the usual stochastic resonance phenomena as in Figs. 2 and 3 can also be found for some certain values of parameters. In a word, the asymmetry of the dichotomous resistor is not the necessary condition for the appearance of phenomena reported in this paper.

Dichotomous noise can often emerge in condensed matters, biological systems, and chemical systems. For the first example, we consider the semiconductor. We assume that the bound energy band of the electrons is $\epsilon_0(T,k,m)$, in which $k=p/\hbar$, p is the momentum, T the temperature, and $m=1,2,\ldots,M$; the first excitation energy band is $\epsilon_1(T,k,n)$ in which $n=1,2,\ldots,N$; the transition rate from $\epsilon_0(T,k,m)$ to $\epsilon_1(T,k,n)$ and vice versa are, respectively, γ_{mn} and γ'_{nm}. It is clear that the transition of the electrons from $\epsilon_0(T,k,m)$ to $\epsilon_1(T,k,n)$ and vice versa constitutes just one asymmetric dichotomous noise. For the second example, we consider the proteins motor. For the protein motor, its fluctuating potential is just one asymmetric dichotomous noise [17]. For the third example, we consider a reversible chemical reaction $A \rightleftharpoons_{k_1}^{k_2} B$. In this reaction, the concentrations of A and B are variable: we set them as x and y. Then the rate equations can be written as $dx/dt = -k_1 x + k_2 y$ and $dy/dt = -k_2 y + k_1 x$, in which $x + y = N_0 = \text{const}$, which is the total concentration of A and B. By defining $\rho_1(x) = x/N_0$ and $\rho_2(y) = y/N_0$, the equations become $d\rho_1(x)/dt = -k_1\rho_1(x) + k_2\rho_2(y)$ and $d\rho_2(y)/dt = -k_2\rho_2(y) + k_1\rho_1(x)$, where $\rho_1(x) + \rho_2(y) = 1$. Here $\rho_1(x)$ and $\rho_2(y)$ are the probability densities of A and B. It is clear that the $\rho_1(x)$ and $\rho_2(y)$ equations are just the master equation for the probability densities of the following multiplicative asymmetric dichotomous noise: This noise takes the values x and y; the transition rates from x to y and vice versa are k_1 and k_2, respectively. Thus, we can say that the reversible chemical reaction $A \rightleftharpoons_{k_1}^{k_2} B$ is just also another asymmetric dichotomous noise.

As a conclusion, we have found strong evidence for the existence of a phenomenon of stochastic giant resonance, the appearance of which results from the existence of a zero noise power spectrum with the nonzero signal power spectrum for the same parameters. As a possible direction for further research, it will be very interesting to study whether the phenomenon of stochastic giant resonance exists in the semiconductor, the protein motor, and the reversible chemical reaction, when inputting periodic signals in these systems (there is the asymmetric dichotomous noise in these systems). Here, it should be mentioned that "stochastic resonance" usually denotes a phenomenon different from the three resonances reported in the present paper. The SNR gets a maximum (or more) for a finite value of the noise intensity [and thus its counterintuitive character (see, for instance, the first article in Ref. [1])]. Here the resonances are obtained for finite values of the frequency of the external signal (including the giant) and noise correlation time. Finally, it is necessary to give further clarification of the physical consequence of the phenomenon of "stochastic giant resonance" reported by us in this paper: The phenomenon of stochastic giant resonance does not imply a maximum in the output intensity, but in the SNR.

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APPENDIX A

w_1, w_2, w_3, w_4, G_5, and G_6 in Eq. (5) are given in this appendix. They are

$$w_1 = G_1 \cos\phi - G_2 \sin\phi, \quad w_2 = G_1 \sin\phi + G_2 \cos\phi,$$

$$w_3 = G_3 \cos\phi - G_4 \sin\phi + G_7,$$

$$w_4 = G_3 \sin\phi + G_4 \cos\phi - G_8, \quad G_5 = m_3 V,$$

$$G_6 = S_2 V + Q_1,$$

with

$$G_1 = \frac{1}{2} A \frac{f_1 h_3 - f_2 h_4}{f_1^2 + f_2^2}, \quad G_2 = -\frac{1}{2} A \frac{f_1 h_4 + f_2 h_3}{f_1^2 + f_2^2},$$

$$G_3 = \frac{1}{2} A \frac{q_1 g_1 - a\omega g_2}{g_1^2 + g_2^2}, \quad G_4 = -\frac{1}{2} A \frac{q_1 g_2 + a\omega g_1}{g_1^2 + g_2^2},$$

$$G_7 = \frac{1}{2} \frac{q_2 g_1}{g_1^2 + g_2^2},$$

$$G_8 = \frac{1}{2} \frac{q_2 g_2}{g_1^2 + g_2^2}, \quad S_2 = \frac{(g - g_0)\frac{U_0}{L} + \frac{U_0}{L}(-R - g)}{\frac{c^2}{L} + \lambda(g - g_0) + \left(\lambda - \frac{R+g}{L}\right)(R+g)},$$

$$Q_1 = \frac{\frac{U_0^2}{L}}{\frac{c^2}{L} + \lambda(g - g_0) + \left(\lambda - \frac{R+g}{L}\right)(R+g)},$$

$$g_1 = L\omega^2 - \lambda^2 L + \lambda[\lambda L + 2(R+g)] + \left[\frac{c^2}{L} + \lambda(g - g_0) - \left(\lambda + \frac{R+g}{L}\right)(R+g)\right],$$

$$g_2 = -2L\lambda\omega + \omega[\lambda L + 2(R+g)],$$

$$q_1 = \frac{a}{L}(-R - g)(g - g_0)\frac{a}{L},$$

$$q_2 = \frac{a^2}{L}, \quad f_1 = L\omega^2 + \frac{c^2}{L} + \lambda(g - g_0) - \left(\lambda + \frac{R+g}{L}\right)(R+g),$$

$$f_2 = [\lambda L + 2(R+g)]\omega,$$

$$h_3 = -\left(\lambda + \frac{R+g}{L}\right)a - (g - g_0)\frac{a}{L}, \quad h_4 = -a\omega,$$

$$m_3 = -\frac{\left(\lambda + \frac{R+g}{L}\right)U_0 + \frac{U_0}{L}(g - g_0)}{\frac{c^2}{L} + \lambda(g - g_0) - \left(\lambda + \frac{R+g}{L}\right)(R + g_0)},$$

$$c_1 = \frac{f_1 h_3 - f_2 h_4}{f_1^2 + f_2^2}, \quad c_2 = \frac{f_1 h_4 + f_2 h_3}{f_1^2 + f_2^2}, \quad \cos\phi = \frac{c_1}{\sqrt{c_1^2 + c_2^2}},$$

$$\sin\phi = \frac{c_2}{\sqrt{c_1^2 + c_2^2}}, \quad A = \sqrt{c_1^2 + c_2^2},$$

$$V=\frac{\lambda U_0+\dfrac{U_0(R+g)}{L}-\dfrac{U_0(g_0-g)}{L}}{\lambda(R+g)+\dfrac{(R+g)^2}{L}-\dfrac{c^2}{L}+\lambda(g_0-g)},$$

$$c=\frac{r_a-r_b}{2},\quad g=\frac{r_a+r_b}{2}.$$

APPENDIX B

a_{10}, a_8, a_6, a_4, a_2, and a_0 in Eq. (9) are given in this appendix. They are

$$a_{10}=8H_4,\quad a_8=8H_5+12\lambda^2H_4,$$

$$a_6=4\lambda^4H_4+12\lambda^2H_5+8H_6+8F\lambda^2H_1,$$

$$a_4=4\lambda^4H_5+12\lambda^2H_6+8H_7+8F\lambda^2H_2+2F\lambda^4H_1,$$

$$a_2=8F\lambda^2H_3+2F\lambda^4H_2+4\lambda^4H_6+12\lambda^2H_7,$$

$$a_0=2F\lambda^4H_3+4\lambda^4H_7,$$

with

$$H_1=(L^2+2Lg_{11}+g_{22}^2)(L^2+2Lf_{11}+f_{22}^2),$$

$$H_2=[f_{11}^2(L^2+2Lg_{11}+g_{22}^2)+g_{11}^2(L^2+2Lf_{11}+f_{22}^2)],$$

$$H_3=f_{11}^2g_{11}^2,\quad H_4=-a^2L^2+q_2L^3,$$

$$H_5=-aL(ag_{11}+q_1g_{22}+af_{11}-h_3f_{22})+Lq_2(2Lf_{11}+f_{22}^2)+q_2g_{11}L^2+(q_1L-ag_{22})(Lh_3+f_{22}a),$$

$$H_6=[f_{11}h_3(q_1L-ag_{22})+q_1g_{11}(Lh_3+f_{22}a)]-(ag_{11}+q_1g_{22})\times(af_{11}-h_3f_{22})+q_2f_{11}^4L+q_2g_{11}(2Lf_{11}+f_{22}^2),$$

$$H_7=q_2g_{11}f_{11}^4,$$

$$g_{11}=-\lambda^2L+\lambda[\lambda L+2(R+g)]+\left[\frac{c^2}{L}+\lambda(-R-g)-\left(\lambda+\frac{R+g}{L}\right)(R+g)\right],$$

$$g_{22}=-L\lambda+2(R+g),$$

$$f_{11}=\frac{c^2}{L}+\lambda(g-g_0)-\left(L+\frac{R+g}{L}\right)(R+g),$$

$$f_{22}=[\lambda L+2(R+g)]^2,\quad F=S_2V+Q_1,$$

in which q_2, h_3, q_1, g, S_2, V, and Q_1 are the same as in Appendix A.

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